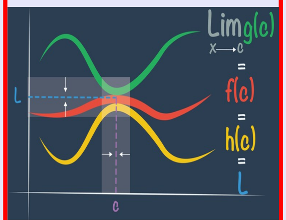
Math 261

Fall 2022

Lecture 18



1) Find $f'(x)$

2) Solve $f'(x) = 0$

$f(x) = x^3 - 12x$

$\boxed{f'(x) = 3x^2 - 12}$

$f'(x) = 0 \rightarrow 3x^2 - 12 = 0$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x + 2 = 0$ OR $x - 2 = 0$

$\boxed{x = -2}$ $\boxed{x = 2}$

1) Find $f'(x)$

2) Solve $f'(x)=0$

3) Find x-values where $f'(x)$ is undefined

$$f(x)=\frac{x-2}{x+1}$$

$$f'(x)=\frac{1\cdot(x+1)-(x-2)\cdot 1}{(x+1)^2}$$

$$f'(x)=\frac{3}{(x+1)^2}$$

$f'(x)=0 \to \frac{3}{(x+1)^2}=0 \to 3\neq 0$ NO Solution

$f'(x)$ undefined $\to \frac{3}{(x+1)^2}$ will be undefined when $(x+1)^2=0$

$x+1=0$

$\boxed{x=-1}$

Find $f'(x)$ for $f(x)=x^2\tan x$

Product Rule

$\boxed{f'(x)=2x\cdot\tan x+x^2\cdot\sec^2 x}$

$\frac{d}{dx}[\tan x]=\sec^2 x$

Find $f'(x)$ for $f(x)=\frac{\cos^2 x}{1-\sin x}$

For now

$\cos^2 x=\cos x\cdot\cos x$

but $\cos^2 x+\sin^2 x=1$

$\cos^2 x=1-\sin^2 x$

$$f(x)=\frac{1-\sin^2 x}{1-\sin x}$$

$$=\frac{(1+\sin x)(1-\sin x)}{1-\sin x}$$

$f(x)=1+\sin x$

$f'(x)=0+\cos x$

$\boxed{f'(x)=\cos x}$

Find $f'(x)$ for $f(x)=\sqrt{x}\,\cos x\,\tan x$

Recall

$\tan x = \dfrac{\sin x}{\cos x}$

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

$f(x)=\sqrt{x}\cdot\cancel{\cos x}\cdot\dfrac{\sin x}{\cancel{\cos x}}$

$f(x)=\boxed{x^{\frac{1}{2}}}\cdot\boxed{\sin x}$

$$f'(x)=\frac{1}{2}x^{\frac{1}{2}-1}\cdot\sin x + x^{\frac{1}{2}}\cdot\cos x$$

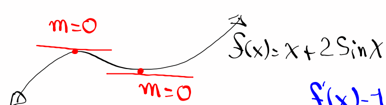
$$=\frac{1}{2}x^{\frac{-1}{2}}\sin x + x^{\frac{1}{2}}\cos x$$

$$=\frac{1}{2x^{1/2}}\sin x + x^{\frac{1}{2}}\cos x$$

$$=\frac{1}{2\sqrt{x}}\sin x+\sqrt{x}\cos x = \frac{\sin x + 2\sqrt{x}\sqrt{x}\cos x}{2\sqrt{x}}$$

$$\boxed{f'(x) = \frac{\sin x + 2x\cos x}{2\sqrt{x}}}$$

For what values of x does the graph of $f(x)=x+2\sin x$ have a horizontal tan. line?



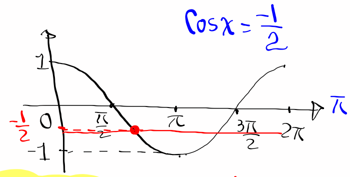
$f(x)=1+2\cdot\cos x$

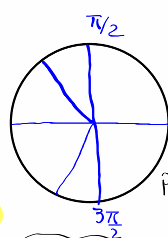
$f'(x)=1+2\cos x$

$f'(x)=0$

$1+2\cos x=0$

$\cos x=\frac{-1}{2}$





Ref. Angle $\cos\alpha=\frac{1}{2}$ $\alpha=\pi/3$

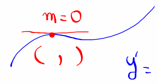
Period for $\cos x$

In QII $\to \pi-\frac{\pi}{3}=\frac{2\pi}{3} \to x=\frac{2\pi}{3}+n\cdot 2\pi$

In QIII $\to \pi+\frac{\pi}{3}=\frac{4\pi}{3} \to x=\frac{4\pi}{3}+n\cdot 2\pi$

n is any integer.

Find points on the curve $y = \frac{\cos x}{2+\sin x}$ at which the tangent is horizontal.



$$y = \frac{\cos x}{2+\sin x}$$

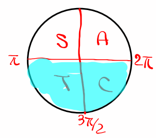
$$y' = \frac{-\sin x(2+\sin x) - \cos x \cdot \cos x}{(2+\sin x)^2}$$

$$y' = \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2}$$

$$y' = \frac{-2\sin x - 1}{(2+\sin x)^2}$$

$y' = 0 \to -2\sin x - 1 = 0$

$\sin x = \frac{-1}{2}$



Ref. Angle

$\sin\alpha = \frac{1}{2} \to \alpha = \frac{\pi}{6}$

QIII $\to \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ $\qquad x = \frac{7\pi}{6} + n \cdot 2\pi$

QIV $\to 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ $\qquad x = \frac{11\pi}{6} + n \cdot 2\pi$

Looking for points

$x = \frac{7\pi}{6} + n \cdot 2\pi$ $\qquad y = \frac{\cos\frac{7\pi}{6}}{2+\sin\frac{7\pi}{6}} = \frac{-\sqrt{3}/2}{2-\frac{1}{2}} = \frac{-\frac{\sqrt{3}}{2}}{\frac{3}{2}}$

$\boxed{y = -\frac{\sqrt{3}}{3}}$

Point

$\left(\frac{7\pi}{6} + n \cdot 2\pi, -\frac{\sqrt{3}}{3}\right)$

$\left(\frac{11\pi}{6} + n \cdot 2\pi, \frac{\sqrt{3}}{3}\right)$

Try the same method for $x = \frac{11\pi}{6} + n \cdot 2\pi$

$$y = \frac{\cos\frac{11\pi}{6}}{2+\sin\frac{11\pi}{6}} = \frac{\frac{\sqrt{3}}{2}}{2-\frac{1}{2}} = \cdots = \frac{\sqrt{3}}{3}$$

Help me expand

$$(A+B)^{10} = A^{10} + 10A^9B + \overset{45}{\boxed{\frac{10 \cdot 9}{2}}} A^8B^2 + \underset{120}{\boxed{\frac{45 \cdot 8}{3}}} A^7B^3 + A^6B^4 + B^{10}$$

$$\frac{120 \cdot 7}{4}$$

$$(A+B)^n = A^n + nA^{n-1}B + \frac{n(n-1)}{2}A^{n-2}B^2 + \frac{n(n-1)(n-2)}{2 \cdot 3}A^{n-3}B^3 + \cdots$$

Prove $\frac{d}{dx}[x^n] = n x^{n-1}$ n is natural #.

$$\frac{d}{dx}[x^n] = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \qquad f(x) = x^n$$

$$= \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^n} + n x^{n-1} h + \frac{n(n-1)}{2} x^{n-2} h^2 + \cdots + h^n \cancel{- x^n}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}\left(n x^{n-1} + \frac{n(n-1)}{2} x^{n-2} h + \cdots + h^{n-1}\right)}{\cancel{h}}$$

$$= \lim_{h\to 0} \left[n x^{n-1} + \cancel{\frac{n(n-1)}{2} x^{n-2} h + \cdots + h^{n-1}}^{\to 0} \right]$$

$$= n x^{n-1}$$

Power Rule $\frac{d}{dx}[x^n] = n x^{n-1}$

Simplify

$$(x - a)(x^{n-1} + x^{n-2} a + x^{n-3} a^2 + \cdots + a^{n-1})$$

$$= \boxed{x^1 \cdot x^{n-1}} + \cancel{x^1 \cdot x^{n-2}} a + \cancel{x^1 \cdot x^{n-3}} a^2 + \cdots + \cancel{x a^{n-1}}$$

$$\cancel{- a x^{n-1}} \cancel{- x^{n-2}} a^2 \cancel{- x^{n-3}} a^3 \cdots - a \cdot a^{n-1}$$

$$= x^n - a^n$$

Recall

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \qquad \text{for } f(x) = x^n$$

$$f'(a) = \lim_{x \to a} \frac{x^n - a^n}{x - a}$$

$$= \lim_{x \to a} \frac{\cancel{(x-a)}(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \cdots + a^{n-1})}{\cancel{x-a}}$$

$$= a^{n-1} + a^{n-2} \cdot a + a^{n-3}a^2 + \cdots + a^{n-1}$$

$$= \underbrace{a^{n-1} + a^{n-1} + a^{n-1} + \cdots + a^{n-1}}_{n \text{ of them}}$$

$$f'(a) = n\,a^{n-1}$$

$$f'(x) = n\,x^{n-1}$$